Anomalies and vacuum effects in strong magnetic fields

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Outline

- Search for particles that couple to $F\tilde{F} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} = 4\vec{E}\cdot\vec{H}$
- Anomalies as origin of the coupling to $F\tilde{F}$
- Models with extra dimensions and their low energy signatures
- Theories with anomaly inflow
- Examples
  - Anomalous electrodynamics
  - Anomalous SM
- Current experimental restrictions
- Anomalies and new vector fields

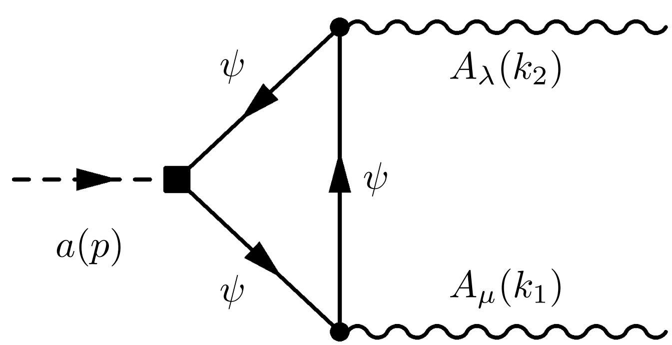
- New symmetry was suggested by Peccei-Quinn as a solution of the **strong CP-problem**
- It implied the existence of the new particle – **axion**

Weinberg'77
Wilczek'77

- More generally, axion-like particles (**ALP**) is a pseudoscalar

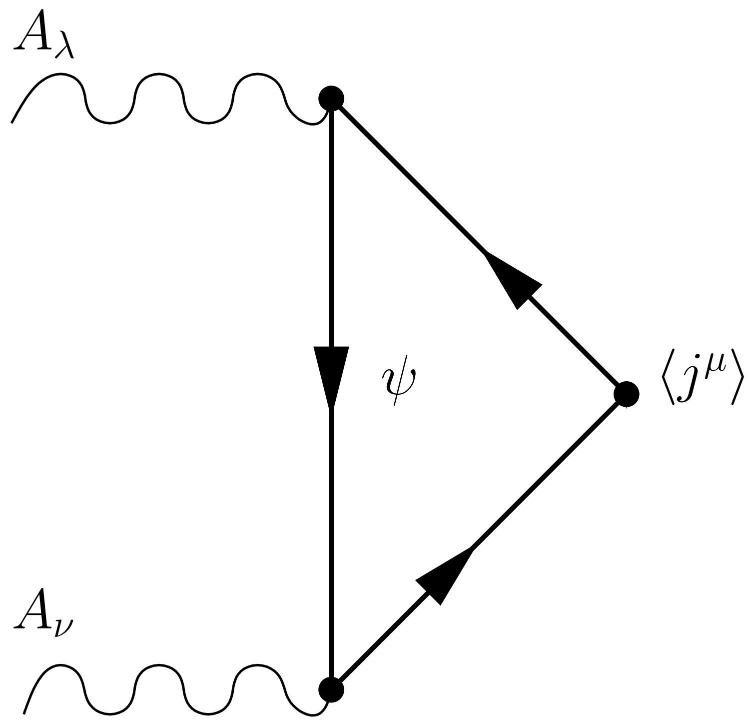
$$\mathcal{L}_{\text{ALP}} = \frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{4M}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

- One can search for ALPs in parallel electric and magnetic fields
- **Origin** of $a\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$?



- Triangular fermionic loops give rise to **anomalies** – violation of a classical symmetry at the quantum level.

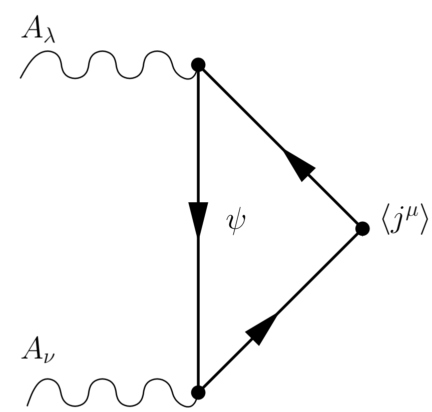
- **Anomaly of a global symmetry** leads to new physics (**ABJ anomaly:** decay of $\boldsymbol{\pi^0 \longrightarrow 2\gamma}$, axion coupling to $\boldsymbol{F\tilde{F}}$)



$$\partial_\mu \langle j^\mu \rangle = \frac{e^3}{16\pi^2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$$

- The same type of diagram can lead to non-conservation of the *gauge current* – **gauge anomaly**

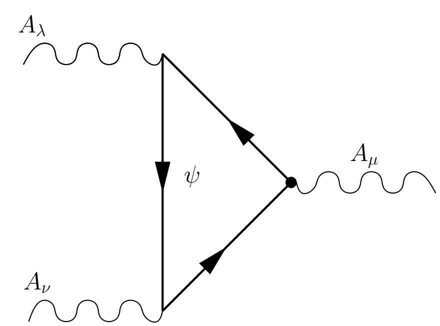
- If a theory contains **chiral fermions** it can happen that gauge symmetry of classical theory is violated by quantum corrections



$$\partial_\mu \langle j^\mu \rangle = \frac{e_L^3 - e_R^3}{16\pi^2} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} = \frac{e_L^3 - e_R^3}{4\pi^2} (\vec{E} \cdot \vec{B})$$

- **Anomaly of gauge symmetry** leads to the loss of unitarity in a theory:

  $k^\mu A_\mu \neq 0$ – **longitudinally polarized** photon appears in the spectrum



Gauge anomalies make theory inconsistent

- Maxwell equations need conserved current

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \quad \partial_\mu j^\mu = 0$$

.

- Normally current conservation is guaranteed by gauge symmetry.
- On quantum level one has

$$\partial_\mu F^{\mu\nu} = \langle j^\nu \rangle$$

- If $\partial_\mu \langle j^\nu \rangle \neq 0$ – the theory becomes inconsistent

Standard Model does contain chiral fermions.

How such a theory can be consistent?

Anomaly cancellation in SM

- Several chiral fermions can help each other: $\partial_\mu \langle j^\mu_\psi + j^\mu_\chi \rangle = 0$
- It may happen that one group of chiral fermions is much heavier than the other ($m_\psi \ll m_\chi$).
- **Example:** $m_{\rm bottom} \sim 5$ GeV $\ll m_{\rm top} \sim 174$ GeV. However, SM *without* t-quark is **anomalous** – gauge invariance is broken at quantum level and the theory would lose unitarity.
- How does cancellation works at energies $m_\psi \ll E \ll m_\chi$?

D'Hoker-Farhi current

- Usual logic of effective field theories tells us that contributions from heavy particles are suppressed as powers of E/m_χ ("Decoupling theorem") Appelquist, Corazzone'75
- Terms like $\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$ have dimensionless couplings – do not depend on mass of fermions producing them.
- Heavy **chiral** fermions can produce quantum corrections to the current, not suppressed by their mass D'Hoker-Farhi'84

$$j^\mu_{\text{DF}} \sim \epsilon^{\mu\nu\lambda\rho}\frac{\phi^* \overleftrightarrow{D}_\nu \phi}{|\phi|^2}F_{\lambda\rho}$$

ϕ – **Higgs field**. This current survives even as $|\phi| \to \infty$

- D'Hoker-Farhi current is not conserved:

$$\partial_\mu j^\mu_{\text{DF}} \sim \epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

Observational signatures of anomalies

- Anomaly analysis gives information about the high energy physics
- For example, the discovery of b-quark strongly hinted at existence of the t-quark (no matter how heavy it would be)!
- Can the anomalous currents a là D'Hoker-Farhi, produced by some heavy particles, be observed at low energy?
- There are **2 possibilities**
  - Sum of anomalous currents cancels: $j_\psi + j_\chi = 0$
  - Sum of **divergences** of anomalous currents cancels: $\partial_\mu \langle j^\mu_\psi + j^\mu_\chi \rangle = 0$, while $j_\psi + j_\chi \neq 0$

Example: higher-dimensional current

- Consider a theory of 4+1 dimensional fermions interacting with a topological defect.

$$S = \int d^4x\, dz \sum_{f=1}^{2} \bar{\Psi}_f(x)\Big(i\not{D}_f + m_f(z)\Big)\Psi_f(x)\,.$$

  The fermionic mass has a “kink-like” structure in the 5th direction

- Fermions in the bulk are vector-like and massive. Chiral zero modes propagate only in 3+1

- At low energies there are only chiral zero modes, which produce 4-dimensional anomalous current

- **How the gauge invariance of full 5-dimensional theory is restored?**

Anomaly inflow

- Massive bulk modes produce a current, flowing towards the brane:

Faddeev, Shatashvili'84 Callan, Harvey'85

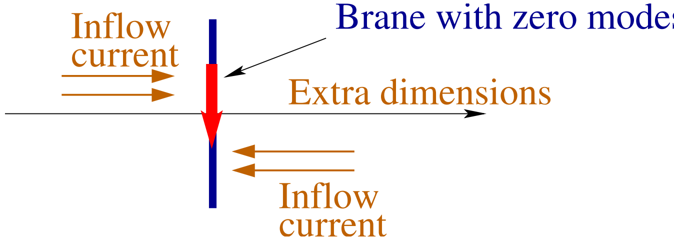
$$J^z \sim \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$$

– **anomaly inflow** current

- Similarly to the D'Hoker-Farhi current it is not suppressed by the mass of bulk fermions

- $\partial_z J^z + \partial_\mu j^\mu_{\text{z.m.}} = 0$, but obviously $J + j_{\text{z.m.}} \neq 0$

This current corresponds to the **Chern-Simons** term in the 5-dimensional effective action:



$$\int A \wedge F \wedge F = \int \epsilon^{abcde} A_a F_{bc} F_{de}$$

Topological term. Does not contain metric and dimensionful constants

Anomalous Extensions of SM

- Choice of hypercharges in SM is controlled by Yukawa interaction.
- This fixes hypercharge assignments up to two constants: κ_l in lepton sector and κ_q in quark sector.
-

e_L	e_R	ν_L	Q_L	u_R	d_R	ν_R
$-1+\kappa_l$	$-2+\kappa_l$	$-1+\kappa_l$	$\frac{1}{3}+\kappa_q$	$\frac{4}{3}+\kappa_q$	$-\frac{2}{3}+\kappa_q$	κ_l

- These constants are usually chosen to be zero to ensure that SM is anomaly free:

$$\partial_\mu j^\mu_Y = \frac{\mathrm{Tr}[Y^3]}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}_Y F^{\lambda\rho}_Y + \frac{\mathrm{Tr}[Y_L]}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}\,\mathrm{Tr}_{SU(2)}\,G_{\mu\nu}G_{\lambda\rho}$$

  where $\mathrm{Tr}[Y^3] = 6(\kappa_l + 3\kappa_q)$ and $\mathrm{Tr}[Y_L] = -2(\kappa_l + 3\kappa_q)$
- Experimentally $\kappa_l + 3\kappa_q = \frac{e-p}{e} < 10^{-21}$
- This number is small but may be **non-zero** if SM is a sector a bigger theory. For example, a theory with extra dimensions

Vector-like Electrodynamics

- Arbitrary choice of parameters κ_l, κ_q leads to anomaly of **hypercharge current**.
- However, for **any** choice of hypercharges, electrodynamics remains vector-like and **anomaly-free**
- If SM is expanded by some additional **4-dim fields**, it may happen that the electrodynamics will also become chiral
- If the theory contains additional U(1) **4-dim fields**, there can **mixed anomaly**. These **anomalies** of SM can be canceled by inflow from extra dimensions
- What are the consequences of the presence of inflow currents from the point of view of the low energy physics on a brane?

Manifestations of Anomaly Inflow?

- Experimentally electric neutrality of matter is confirmed to a very high precision ($\frac{e-p}{e} < 10^{-21}$)

- What if still $\frac{e-p}{e} \neq 0$?

- What will the 4-dimensional observer detect?

  – Flux of particles from higher dimensions? $\leftarrow$ **Wrong!**

  – Five-dimensional transversal photon:
  $k^\mu A_\mu + k^5 A_5 = 0$ but $k^\mu A_\mu \neq 0$? $\leftarrow$ **Wrong!**

- The inflow current is a **vacuum** current – not carried by real particles. It is caused by a redistribution of the charges in the Dirac sea of the full theory, leads to an appearance of an electric charge on the brane.

Anomalous Electrodynamics

Consider again our simplest example of anomalous electrodynamics on a domain wall in 4+1 dimensions (z – **coordinate of 5th dim**)

$$S = \underbrace{-\frac{1}{4e_5^2}\int \Delta(z) F\wedge\star F}_{\text{5-dim kinetic term}} + \underbrace{\frac{1}{4}\int \kappa(z) A\wedge F\wedge F}_{\text{Anomaly inflow interaction}} + \underbrace{\int d^4x\, \mathcal{L}_{matter}}_{\text{Anomalous theory: } \partial_\mu j^\mu \sim F\tilde{F}}$$

Oda 2000; Dubovsky et al. 2000; Shaposhnikov, Tinyakov 2001

Factor $\Delta(z) = \exp(-2M|z|)$ is responsible for localization of the gauge fields on a brane

Without CS this action would describe a 4-dim theory for $E < M$

Normalizable zero mode of gauge fields: $\partial_z F_{\mu\nu} = 0, \quad F_{\mu z} = 0$

Set of **non-linear** 5-dimensional Maxwell-like equations:

$$\partial_b\Big(\Delta(z)F^{\mu b}\Big) = e_5^2\Big(J^\mu_{\text{CS}} + j^\mu_{\text{SM}}\Big) \qquad a,b = 0,\ldots,4$$

$$\Delta(z)\partial_\mu F^{z\mu} = e_5^2 J^z_{\text{CS}} \qquad \mu = 0,\ldots,3.$$

$$J^\mu_{\text{CS}} = 3\kappa(z)\epsilon^{\mu\nu\lambda\rho}F_{z\nu}F_{\lambda\rho}$$

$$J^z_{\text{CS}} = \frac{3}{4}\kappa(z)\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

Inflow current J_{CS} **cancels anomaly on the brane:**

$$\partial_\mu J^\mu_{\text{CS}} + \partial_z J^z_{\text{CS}} + \partial_\mu j^\mu_{\text{SM}} = 0$$

Plane wave propagating in the strong magnetic field $H_x \approx \mathrm{const}$ and $\kappa_0 \ll 1$. For the wave with parallel to $\vec{H}$ polarization

Boyarsky, O.R 2007

$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z A_x\Big) + \Box A_x = \underbrace{\frac{\alpha_{\text{EM}}^2 \kappa_0^2 \vec{H}^2}{M_5^2 \Delta^2(z)}}_{\text{CS term, non-perturbative in } \kappa_0!} A_x + \mathcal{O}(\kappa_0)$$

- ⋆ **Massive** wave equation $\Box A_x(x) - m_{\gamma H}^2 A_x(x) = 0$
- ⋆ Mass $m_{\gamma H}^2 \sim \alpha_{\text{EM}} \kappa_0 |\vec{H}|$ depends only on 4-dim quantities. It is not suppressed by the scale of 5th dimension M_5
- ⋆ Massless wave equation for perpendicular to the magnetic field component $\Box A_y(x) = 0$
- ⋆ This leads to the **ellipticity** (birefringence) of the linearly polarized light $\Delta\phi = \frac{m_{\gamma H}^2}{2\omega} L \sim \frac{\kappa_0 \alpha_{\text{EM}} |\vec{H}|}{2\omega} L$

Ellipticity in anomalous electrodynamics

- Ellipticity also appears in theories where photon interacts with ALPs, millicharged particles, etc. or due to the QED corrections to the electrodynamics Lagrangian

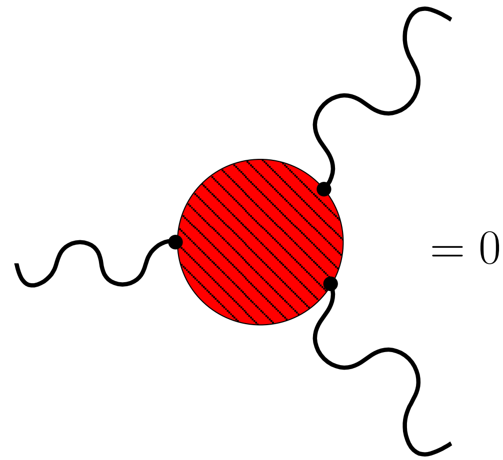
- Signatures of anomalous electrodynamics **differ** from these examples

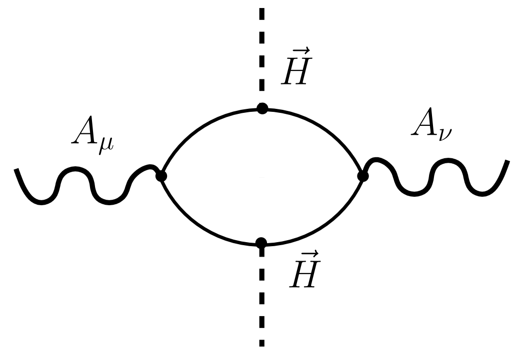
- Unlike theories with ALP, here there is **no** dichroism (rotation of polarization plane) in this theory, as there are **no new light degrees of freedom**. There is also no “light shining through the wall”

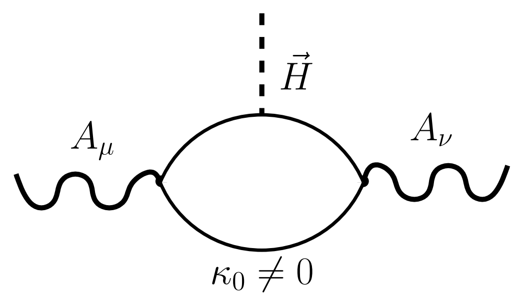
- Ellipticity in our case is proportional to the $|\vec{H}|$ (unlike QED or ALP cases, where ellipticity $\sim \vec{H}^2$). This is a **signature** of non-local (higher-dimensional) physics

- The dependence of ellipticity on the optical path L is linear (unlike in the theories with ALPs)

- **Furry theorem in QED:** any diagram with odd number of external photon legs is zero (CP-symmetry).
- **QED** corrections to Maxwell theory
  $\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \frac{14\,\alpha^2_{\text{EM}}}{45\,m_e^4}(\vec{E}\vec{H})^2 + \ldots$
- The Euler-Heisenberg Lagrangian gives ellipticity but does not lead to the photon mass. Static (capacitor) experiment would give no results
- Once $\kappa_0 \neq 0$ there is no **Furry theorem** as $e_L \neq e_R$. Anomalous triangular diagram exists and leads to a pole in the photon propagator with $m^2_{\gamma H} \sim \kappa_0 \alpha_{\text{EM}} |\vec{H}|$







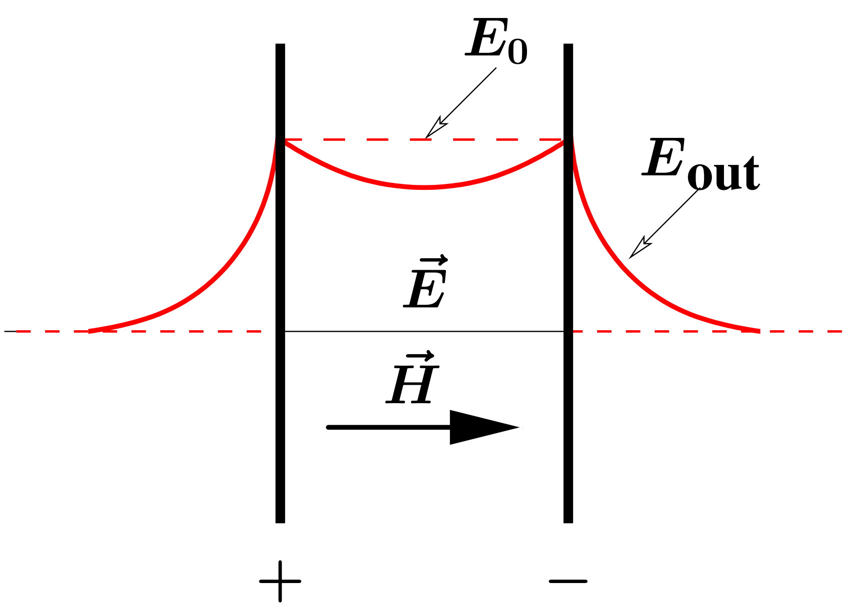
There is also another experimental setup, which can observe anomaly inflow and distinguish 5-dimensional theory from its 4-dimensional counterparts

Static solution in magnetic field

Static solution in strong magnetic field $\vec{H} \approx \text{const}$ and $\kappa_0 \ll 1$ to one equation for the electrostatic potential $\Phi(x, z) = \phi(x)\chi(z)$:

Boyarsky, O.R., Shaposhnikov PRD 2005

$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z\Phi\Big) + \vec{\nabla}^2\Phi = \underbrace{\frac{\alpha_{\text{EM}}^2\,\kappa_0^2\,\vec{H}^2}{M_5^2\,\Delta^2(z)}\Phi}_{\text{CS term, non-perturbative in }\kappa_0} + \underbrace{e_5^2\rho(x)\delta(z)}_{\text{source charge}} + \mathcal{O}(\kappa_0)$$



- ⋆ Effective Poisson equation:
- ⋆ $\vec{\nabla}^2\phi(x) - m_{\gamma H}^2\phi(x) = \alpha_{\text{EM}}\rho(x)$
- ⋆ Electric field **gets screened** as if **photon** had **become massive**
- ⋆ Mass $m_{\gamma H}^2 = \alpha_{\text{EM}}\kappa_0|\vec{H}|$ depends **only** on 4-dim quantities – fine-structure constant α_{EM} and magnetic field $\vec{H}$ as measured on the brane.

Anomalous $\gamma\gamma Z$ Coupling

In SM there can be only $\gamma\gamma Z$ anomalies. ***The analysis gets messy***

$$\partial_\mu j^\mu_Z = -\frac{4N_f\,(\kappa_l + 3\kappa_q)}{\pi^2 \sin 2\theta_W}\vec{E}_\gamma\cdot\vec{H}_\gamma\ ;\ \ \partial_\mu j^\mu_\gamma = -\frac{8N_f(\kappa_l + 3\kappa_q)}{\pi^2 \sin 2\theta_W}(\vec{E}_\gamma\cdot\vec{H}_Z + \vec{E}_Z\cdot\vec{H}_\gamma)$$

- A background (capacitor) with $\vec{E}\cdot\vec{H} \neq 0$ creates an
  inflow of Z current
- Anomalous density of Z charge creates Z field and
  non-trivial γZ background
- Non-trivial γZ background leads to
  inflow of electro-magnetic current
- Anomalous distribution of electric charge on the brane is created and electric field is modified **as if photon has acquired mass**

$$m^2_{\gamma H} = \frac{2N_f e_4^2|\vec{H}|}{\pi^2 \sin 2\theta_W}(\kappa_l + 3\kappa_q)$$

does not depend on m_Z or M_5!

Setup of static experiment

- In the SM $\kappa \lesssim 10^{-21}$ which leads to the $m_{\gamma H} \lesssim 10^{-10}$ **eV** for the magnetic field 10 Tesla.

- **Idea N° 1:** if one "turns on" mass for the photon, the capacitance of a system would change $\Rightarrow$ Create an RC-circuit, turn on strong magnetic field and measure the shift of capacitance. The change of capacitance $\frac{\Delta C}{C} \sim m_{\gamma H}$. Possible to measure shift of capacitance with **femto**Farad (10^{-3}pF) precision and thus masses $m_{\gamma H} \gtrsim 10^{-8}$ eV

- **Idea N° 2:** Attraction force between two charged parallel plates (ideal capacitor) can be measured with **nano**Newton precision. Can probe mass range $m_{\gamma H} \gtrsim 10^{-11}$ eV.

- Tentative limit on measurements of deviation from the Gauss law $\sim 10^{-14}$ eV

- Unique signature $m_{\gamma H} \sim \sqrt{|\vec{H}|}$

Example 2: New vector field and CS terms

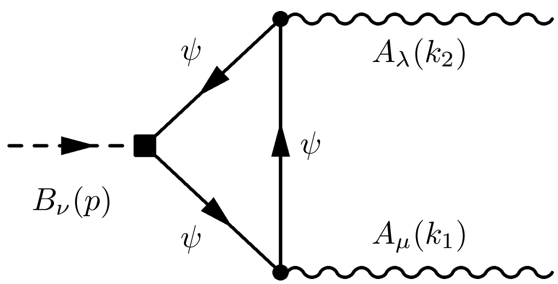
- In the SM model fermions have both vector and axial couplings to gauge fields (e.g. $e^{\pm}$ interaction with electromagnetic and Z field)

- Imagine an extension of the SM where some fermions (either SM or new ones) interact with both photon A_μ and new gauge field B_μ

Antoniadis, Boyarsky, O.R 2006

- Anomalous triangular diagram induces **4-dim** Chern-Simons-like coupling between two fields:

$$\mathcal{L}_{\text{CS}} = \kappa\epsilon^{\mu\nu\lambda\rho} A_\mu B_\nu \partial_\lambda A_\rho$$



- We obtain an effective theory

$$\mathcal{L} = -\frac{1}{4}|F_A|^2 - \frac{1}{4}|F_B|^2 + \frac{m_B^2}{2}|d\theta + B|^2 + \kappa A\wedge B\wedge F_A + \kappa\theta F_A\wedge F_A$$

Longitudinal component at low energies

$$\mathcal{L} = -\frac{1}{4}|F_A|^2 - \frac{1}{4}|F_B|^2 + \frac{m_B^2}{2}|d\theta + B|^2 + \kappa A\wedge B\wedge F_A + \kappa\theta F_A\wedge F_A$$

- The theory is gauge invariant with respect to variation of the $B_\mu \to B_\mu + \partial_\mu\lambda$ and $\theta = \theta - \lambda$.
- However, B_μ couples to the current which is **not conserved**:

$$J_B^\mu = \frac{\delta\mathcal{L}}{\delta B_\mu} = \kappa A\wedge F_A; \qquad \partial_\mu J_B^\mu = \kappa F_A\wedge F_A$$

- Longitudinal component of the B-field **does not decouple** and **behaves as ALP** with mass m_B and coupling $M_{\text{ALP}} = \frac{m_B}{\kappa}$

Longitudinal component at high energies

- If there is an additional massive particle (with **mass** m_0), interacting with A_μ and B_μ, for $E > m_0$ effective Lagrangian becomes non-local
- Now B_μ couples to the **conserved** current

Antoniadis, Boyarsky, O.R., to appear

$$J_B^\mu = \frac{\delta \mathcal{L}}{\delta B_\mu} = \kappa A \wedge F_A + \kappa \frac{\partial}{\Box} F_A \wedge F_A$$

- For example, fermions with mass m_0 will produce the following term in the effective action

$$\mathcal{L}_\psi = \kappa \left(\theta \frac{m_0^2}{\Box + m_0^2} - \partial_\mu B^\mu \frac{1}{\Box + m_0^2} \right) F_A \wedge F_A$$

- At energies $E \gtrsim m_0$ the production of the longitudinal polarization is suppressed as $(m_B/E)^2$
- If 1 eV $< m_0 < 1$ keV we will have effects in laboratory but not in stars!

Conclusion

- There is a wide range of models where anomaly analysis predicts new phenomena in parallel electric and magnetic fields.
- Some of these models predict effects in strong magnetic fields, but **do not** introduce **new light particles**. Thus no stellar constraints, no contradictions to **CAST** bounds, etc.
- Experiments (such as PVLAS, ALPS, OSQAR, BMV, . . .) can also **probe for the signatures** of these theories (and e.g. discover extra dimensions!)
- Apart from optical experiments (measuring ellipticity and dichroism) and “light shining through the wall”, there is an alternative approach to probe for these theories – **static** “capacitor” experiment

Thank you for your attention!

The End

Anomalies in SM on D-branes

- Appearance of additional **anomalous U(1) groups** is a generic feature in D-brane constructions of SM

Ibanez, Rabadan, Uranga'98

- Anomalous parameter can have **arbitrary** values

Antoniadis, Kiritsis, Rizos'02

- Effects, similar to those, appearing in SM can be induced via anomalous $\gamma\gamma\gamma'$ **coupling**.

Antoniadis, Boyarsky, O.R in progress

- This may produce the low-energy string theory signature not suppressed by string scale M_s?!

Conclusion

- In theories with **anomaly inflow** the electric charge, placed in a magnetic field, gets screened. This **low-energy** effect can serve as a **signature of extra dimensions.**
- Modern experimental data shows that our world is non-anomalous with a very high precision. However, with these restrictions in mind **the effect can be pronounced enough to be detected.**
- Any higher-dimensional theory should either present a mechanism ensuring that the brane world is non-anomalous or explain a **fine-tuning** of the hypercharges.
- Anomalous U(1) couplings generically appear in string vacua. Possible **experimental tests of string theory**?

Static solution

Five coupled non-linear equations reduce for $\vec{H} \approx \mathrm{const}$ and $\kappa_0 \ll 1$ to one equation for the electrostatic potential $\Phi(x,z) = \phi(x)\chi(z)$:

$$\partial_z\Big(\Delta(z)\partial_z\Phi\Big)+\Delta(z)\vec{\nabla}^2\Phi = \underbrace{\frac{\alpha_{\text{EM}}^2\kappa_0^2\vec{H}^2}{M_5^2\Delta(z)}\Phi}_{\text{CS current}} + \underbrace{e_5^2 q(x)\delta(z)}_{\text{source charge}} + \mathcal{O}(\kappa_0)$$

non-perturbative in κ_0!

$$\chi(z) \approx \exp\left(-\frac{m_{\gamma H}^2}{M_5^2}e^{2M_5|z|}\right) \qquad \vec{\nabla}^2\phi(x) - m_{\gamma H}^2(x) = \alpha_{\text{EM}} q(x)$$

$$m_{\gamma H}^2 = \alpha_{\text{EM}}\kappa_0|\vec{H}|$$

Go back...

BULK THEORY

A model of localization of both fermions and gauge fields.

$$S = -\frac{1}{4e_5^2}\int d^5x\, \Delta(z) F_{ab}^2 + \int d^5x \sum_{f=1}^{2} \bar{\Psi}_f(x)\Big(i\not{D}_f + m_f(z)\Big)\Psi_f(x).$$

There are two fermions $\Psi_{1,2}$, interacting with the gauge field with the different charges: $\not{D}_f = \not{\partial} + \frac{e_f}{e_5}\not{A}$, $e_1 \neq e_2$. The fermionic mass terms $m_1(z) = -m_2(z)$ have a “kink-like” structure in the direction z: $m_1(z \to \pm\infty) \to \pm m_\psi$.

MASSES FOR FERMIONIC ZERO MODES

The only way to make the electro-dynamics anomalous is to take left and right moving fermions with different electric charges. Thus one can only introduce a mass term via the Higgs mechanism with an electrically charged Higgs field:

$$S_\phi = \int d^5x \left[\left|D_a\phi\right|^2 - m_\phi^2(z)|\phi|^2 - \frac{\lambda}{4}|\phi|^4 + f\bar{\Psi}_1\Psi_2\phi + \mathrm{h.c.}\right] ,$$

where $D_\mu\phi = i\partial_\mu\phi + (\frac{e_L}{e} - \frac{e_R}{e})A_\mu\phi$ and the Higgs mass $m_\phi^2(z)$ is negative at $z = 0$ and tends to the positive constant in the bulk, as $|z| \to \infty$.

EFFECTIVE FIELD THEORIES AND "DECOUPLING THEOREM"

The usual logic behind **effective field theories**: integration of massive fields only leads to renormalization of charges and fields, while all additional interaction suppressed by some positive power of E/M.**["Decoupling theorem" Appelquist,Corazzone'75]**

Question: if the mass scale of extra dimensions is much bigger than our present energies — can one still expect to see any low energy signatures?

Yes! The "decoupling theorem" does not always hold. The most famous counterexample: theories, with **Chern-Simons-like** interactions.**[Redlich'83]**

- In 2+1 dimensions:

$$\log\det(i\gamma^\mu\partial_\mu + M + e\gamma^\mu A_\mu) = \frac{e^2}{8\pi^2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \ldots$$

- Chern-Simons term survives even as $M \to \infty$!
- True in any **odd** space-time dimensions.
- **What about 3+1 dimensions?**

$U(1)^3$ AND $U(1) \times SU(2)^2$ ANOMALY

$$U(1)^3 \;:\; \partial_\mu j^\mu_Y = \frac{\mathrm{Tr}[Y^3]}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}_Y F^{\lambda\rho}_Y + \frac{\mathrm{Tr}[Y_L]}{16\pi^2}\epsilon^{\mu\nu\lambda\rho}\,\mathrm{Tr}_{SU(2)}\,G_{\mu\nu}G_{\lambda\rho}$$

$$U(1) \times SU(2)^2 \;:\; D^\mu j^\alpha_\mu = \frac{\mathrm{Tr}[Y_L]}{8\pi^2}\epsilon^{\mu\nu\lambda\rho}G^\alpha_{\mu\nu}F_{\lambda\rho}$$

SIGNATURES OF EXTRA DIMENSIONS

- New particles ("KK towers") appear. SM particles disappear into bulk. **High-energy signatures:** only at energies above the **mass gap**.

- Certain theories lead to a modification of Newtons's law at sub-mm scales – **low-energy signature**[Arkani-Hamed,Dimopoulos,Dvali'98]

- Theories with **anomaly inflow:** special type of brane-bulk interaction, **not suppressed** by a mass gap. Low-energy signatures? [This talk]

...and things got messy...

Equations for γ field

$$\left\{\begin{aligned}
\partial_z\Big(\Delta(z)\partial_z\Phi_\gamma\Big) + \Delta(z)\nabla^2\Phi_\gamma &= -e_5^2\Big(q(x)\delta(z) + j^0_{\mathsf{DF}} + J^0_{\mathsf{CS},\gamma}\Big)\,,\\
\partial_z\Big(\Delta(z)F^{xz}\Big) + \frac{\Delta(z)}{r}\partial_r\Big(rF^{xr}\Big) &= e_5^2\Big(j^x_{\mathsf{DF}} + J^x_{\mathsf{CS},\gamma}\Big)\,,\\
\partial_z\Big(\Delta(z)F^{rz}\Big) + \Delta(z)\partial_x F^{rx} &= e_5^2\Big(j^r_{\mathsf{DF}} + J^r_{\mathsf{CS},\gamma}\Big)\,,\\
\partial_z\Big(\Delta(z)F^{\theta z}\Big) + \Delta(z)\partial_x F^{\theta x} + \frac{\Delta(z)}{r}\partial_r\Big(rF^{\theta r}\Big) &= e_5^2\Big(j^\theta_{\mathsf{DF}} + J^\theta_{\mathsf{CS},\gamma}\Big)\,,\\
\Delta(z)\Big(\partial_x F^{xz} + \frac{1}{r}\partial_r\left(rF^{rz}\right)\Big) &= -e_5^2 J^z_{\mathsf{CS},\gamma}\,,
\end{aligned}\right.$$

Equations for Z field

$$\left\{\begin{aligned}
\partial_z\Big(\Delta(z)\Phi_{\mathsf{Z}}\Big) + \Delta(z)\nabla^2\Phi_{\mathsf{Z}} - e_5^2 m_{\mathsf{Z}}^2(z)\Phi_{\mathsf{Z}} &= -e_5^2\Big(q_{\mathsf{Z}}(x)\delta(z) + j^0_{\mathsf{DF,Z}} + \mathcal{J}^0_{\mathsf{CS,Z}}\Big)\,,\\
\partial_z\Big(\Delta(z)\mathcal{F}^{xz}\Big) + \frac{\Delta(z)}{r}\partial_r\Big(r\mathcal{F}^{xr}\Big) - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^x &= e_5^2\Big(j^x_{\mathsf{DF,Z}} + \mathcal{J}^x_{\mathsf{CS,Z}}\Big)\,,\\
\partial_z\Big(\Delta(z)\mathcal{F}^{rz}\Big) + \Delta(z)\partial_x\mathcal{F}^{rx} - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^r &= e_5^2\Big(j^r_{\mathsf{DF,Z}} + \mathcal{J}^r_{\mathsf{CS,Z}}\Big)\,,\\
\partial_z\Big(\Delta(z)\mathcal{F}^{\theta z}\Big) + \Delta(z)\partial_x\mathcal{F}^{\theta x} + \frac{\Delta(z)}{r}\partial_r\Big(r\mathcal{F}^{\theta r}\Big) - e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^\theta &= e_5^2\Big(j^\theta_{\mathsf{DF,Z}} + \mathcal{J}^\theta_{\mathsf{CS,Z}}\Big)\,,\\
\Delta(z)\Big(\partial_x\mathcal{F}^{xz} + \frac{1}{r}\partial_r\left(r\mathcal{F}^{rz}\right)\Big) + e_5^2 m_{\mathsf{Z}}^2(z)\mathcal{A}^z &= -e_5^2\mathcal{J}^z_{\mathsf{CS,Z}}\,.
\end{aligned}\right.$$

back to $\gamma\gamma Z$

Experimental Detection?

Model	κ_0	$m_{\gamma H}$, GeV	τ_0, sec	L_0, cm	E_{out}/E_0
new generation	1	10^{-8}	3×10^{-16}	10^{-5}	0
charged ν	10^{-15}	4×10^{-16}	10^{-8}	3×10^{2}	~ 1
electric neutrality	10^{-21}	4×10^{-19}	10^{-5}	3×10^{5}	$\sim 10^{-3}$
massive γ	10^{-36}	10^{-26}	3×10^{2}	10^{13}	$\sim 10^{-10}$

$\tau_0 \sim 1/m_{\gamma H}$ — characteristic time over which the electric field reaches its final state.

E_{out} — the value of the electric field outside the plates of a capacitor at distances much smaller than $L_0 \sim 1/m_{\gamma H}$.

An initial value of electric field $E_0 \sim 10^7$ Volt/m, magnetic field $H \sim 10^5$ Gauss, the distance between the plates $d = 10^2$ cm.

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